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for the Metalworking Trades

Ulrich Fischer, Roland Gomeringer, Max Heinzler, Roland Kilgus, Friedrich Näher, Stefan Oesterle, Heinz Paetzold, Andreas Stephan

Mechanical and Metal Trades Handbook

3rd English edition

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Authors:

Ulrich Fischer	Dipl.-Ing. (FH)	Reutlingen
Roland Gomeringer	Dipl.-Gwl.	Meßstetten
Max Heinzler	Dipl.-Ing. (FH)	Wangen im Allgäu
Roland Kilgus	Dipl.-Gwl.	Neckartenzlingen
Friedrich Näher	Dipl.-Ing. (FH)	Balingen
Stefan Oesterle	Dipl.-Ing.	Amtzell
Heinz Paetzold	Dipl.-Ing. (FH)	Mühlacker
Andreas Stephan	Dipl.-Ing. (FH)	Marktoberdorf

Editor:
Ulrich Fischer, Reutlingen

Graphic design:
Design office of Verlag Europa-Lehrmittel, Ostfildern, Germany

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The content of the chapter „Program structure of CNC machines according to PAL" (page 412 to 424) complies with the publications of the PAL Prüfungs- und Lehrmittelentwicklungsstelle (Institute for the development of training and testing material) of the IHK Region Stuttgart (Chamber of Commerce and Industry of the Stuttgart region).

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3rd edition, 2012

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All printings of this edition may be used concurrently in the classroom since they are unchanged, except for some corrections to typographical errors and slight changes in standards.

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Preface

The Mechanical and Metal Trades Handbook is well-suited for shop reference, tooling, machine building, maintenance and as a general book of knowledge. It is also useful for educational purposes, especially in practical work or curricula and continuing education programs.

Target Groups
- Industrial and trade mechanics
- Tool & die makers
- Machinists
- Millwrights
- Draftspersons
- Technical Instructors
- Apprentices in above trade areas
- Practitioners in trades and industry
- Mechanical Engineering students

Notes for the user

The contents of this book include tables and formulae in eight chapters, including Tables of Contents, Subject Index and Standards Index.

The **tables** contain the most important guidelines, designs, types, dimensions and standard values for their subject areas.

Units are not specified in the legends for the **formulae** if several units are possible. However, the calculation examples for each formula use those units normally applied in practice.

The **Table of Contents** in the front of the book is expanded further at the beginning of each chapter in form of a partial Table of Contents.

The **Subject Index** at the end of the book (pages 435 – 444) is extensive.

The **Standards Index** (pages 425 – 434) lists all the current standards and regulations cited in the book. In many cases previous standards are also listed to ease the transition from older, more familiar standards to new ones.

Changes in the 3rd edition

In the present edition, we have updated the cited standards and restructured, updated, enhanced or added the following chapters in line with new developments in engineering:

- Fundamentals of technical mathematics
- Strength of materials
- Plastics
- Production management
- Forming
- Welding
- PAL programming system for NC turning and NC milling
- Steel types
- Material testing
- Machining processes
- Injection molding (new)
- GRAFCET

Acknowledgement

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November 2012 Authors and publisher

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Standards and other Regulations

Standardization and standards terms

Standardization is the systematic achievement of uniformity of material and non-material objects, such as components, calculation methods, process flows and services for the benefit of the general public.

Standards term	Example	Explanation
Standard	DIN 7157	A standard is the published work of standardization, e. g. the selection of particular fits in DIN 7157.
Part	DIN 30910-2	Standards can comprise several parts associated with each other. The part numbers are appended to the main standard number with hyphens. DIN 30910-2 describes sintered materials for filters for example, whereas Part 3 and 4 deal with sintered materials for bearings and formed parts.
Supplement	DIN 743 Suppl. 1	A supplement contains information for a standard, however no additional specifications. The supplement DIN 743 Suppl. 1, for example, contains application examples of load capacity calculations for shafts and axles described in DIN 743.
Draft	E DIN 743 (2008-10)	Draft standards are made available to the public for examination and commenting. The planned new version of DIN 743 on load-bearing calculations of shafts and axes, for example, has been published since October 2008 as Draft E DIN 743.
Preliminary standard	DIN V 66304 (1991-04)	A preliminary standard contains the results of standardization, which have not been released as a standard because of certain provisos. DIN V 66304, for example, discusses a format for exchange of standard part data for computer-aided design.
Issue date	DIN 76-1 (2004-06)	Date of publication which is made public in the DIN publication guide; this is the date at which time the standard becomes valid. DIN 76-1, which sets undercuts for metric ISO threads has been valid since June 2004 for example.

Types of standards and regulations (selection)

Type	Abbreviation	Explanation	Purpose and contents
International Standards (ISO standards)	ISO	**I**nternational **O**rganization for **S**tandardization, Geneva (O and S are reversed in the abbreviation)	Simplifies the international exchange of goods and services, as well as cooperation in scientific, technical and economic areas.
European Standards (EN standards)	EN	**E**uropean Committee for Standardization (Comité Européen de **N**ormalisation), Brussels	Technical harmonization and the associated reduction of trade barriers for the advancement of the European market and the coalescence of Europe.
German Standards (DIN standards)	DIN	**D**eutsches **I**nstitut für **N**ormung e.V., Berlin (German Institute for Standardization)	National standardization facilitates rationalization, quality assurance, environmental protection and common understanding in economics, technology, science, management and public relations.
	DIN EN	European standard for which the German version has attained the status of a German standard.	
	DIN ISO	German standard for which an international standard has been adopted without change.	
	DIN EN ISO	European standard for which an international standard has been adopted unchanged and the German version has the status of a German standard.	
	DIN VDE	Printed publication of the VDE, which has the status of a German standard.	
VDI Guidelines	VDI	**V**erein **D**eutscher **I**ngenieure e.V., Düsseldorf (Association of German Engineers)	These guidelines give an account of the current state of the art in specific subject areas and contain, for example, concrete procedural guidelines for the performing calculations or designing processes in mechanical or electrical engineering.
VDE printed publications	VDE	**V**erband **D**eutscher **E**lektrotechniker e.V., Frankfurt (Association for Electrical, Electronic & Information Technologies)	
DGQ publications	DGQ	**D**eutsche **G**esellschaft für **Q**ualität e.V., Frankfurt (German Society for Quality)	Recommendations in the area of quality technology.
REFA sheets	REFA	Association for Work Design, Industrial Organization and Corporate Development REFA e.V., Darmstadt	Recommendations in the area of production and work planning.

1 Mathematics

Quantity	Symbol	Unit	
		Name	Symbol
Lengths	*l*	meter	m

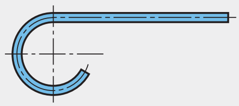
Surface area

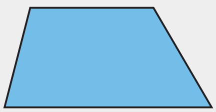
$$A_s = \pi \cdot d \cdot h + 2 \cdot \frac{\pi \cdot d^2}{4}$$

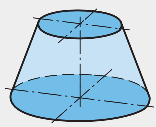
Lateral surface area

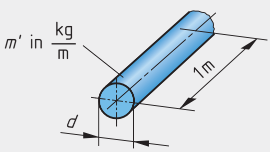
$$A_M = \pi \cdot d \cdot h$$

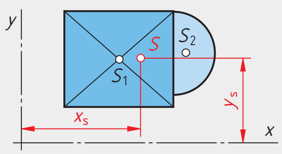
sine	=	**opposite side / hypotenuse**
cosine	=	**adjacent side / hypotenuse**
tangent	=	**opposite side / adjacent side**











M

Units of measurement

SI[1] Base quantities and base units

cf. DIN 1301-1 (2010-10), -2 (1978-02), -3 (1979-10)

Base quantity	Length	Mass	Time	Electric current	Thermo-dynamic temperature	Amount of substance	Luminous intensity
Base units	meter	kilo-gram	second	ampere	kelvin	mole	candela
Unit symbol	m	kg	s	A	K	mol	cd

[1] The units for measurement are defined in the International System of Units SI (**S**ystème **I**nternational d'Unités). It is based on the seven basic units (SI units), from which other units are derived.

Base quantities, derived quantities and their units

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Length, Area, Volume, Angle					
Length	l	**meter**	m	1 m = 10 dm = 100 cm = 1000 mm 1 mm = 1000 µm 1 km = 1000 m	1 inch = 25.4 mm In aviation and nautical applications the following applies: 1 international nautical mile = 1852 m
Area	A, S	square meter are hectare	m^2 a ha	1 m^2 = 10 000 cm^2 = 1 000 000 mm^2 1 a = 100 m^2 1 ha = 100 a = 10 000 m^2 100 ha = 1 km^2	Symbol S only for cross-sectional areas Are and hectare only for land
Volume	V	cubic meter liter	m^3 l, L	1 m^3 = 1000 dm^3 = 1 000 000 cm^3 1 l = 1 L = 1 dm^3 = 10 dl = 0.001 m^3 1 ml = 1 cm^3	Mostly for fluids and gases
Plane angle (angle)	α, β, γ …	radian degrees minutes seconds	rad ° ′ ″	1 rad = 1 m/m = 57.2957…° = 180°/π 1° = $\frac{\pi}{180}$ rad = 60′ 1′ = 1°/60 = 60″ 1″ = 1′/60 = 1°/3600	1 rad is the angle formed by the intersection of a circle around the center of 1 m radius with an arc of 1 m length. In technical calculations instead of α = 33° 17′ 27.6″, better use is α = 33.291°.
Solid angle	Ω	steradian	sr	1 sr = 1 m^2/m^2	An object whose extension measures 1 rad in one direction and perpendicularly to this also 1 rad, covers a solid angle of 1 sr.
Mechanics					
Mass	m	**kilogram** gram megagram metric ton	kg g Mg t	1 kg = 1000 g 1 g = 1000 mg 1 metric t = 1000 kg = 1 Mg 0.2 g = 1 ct	Mass in the sense of a scale result or a weight is a quantity of the type of mass (unit kg). Mass for precious stones in carat (ct).
Linear mass density	m'	kilogram per meter	kg/m	1 kg/m = 1 g/mm	For calculating the mass of bars, profiles, pipes.
Area mass density	m''	kilogram per square meter	kg/m^2	1 kg/m^2 = 0.1 g/cm^2	To calculate the mass of sheet metal.
Density	ϱ	kilogram per cubic meter	kg/m^3	1000 kg/m^3 = 1 metric t/m^3 = 1 kg/dm^3 = 1 g/cm^3 = 1 g/ml = 1 mg/mm^3	The density is a quantity independent of location.

Units of measurement

Quantities and Units (continued)

M

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Mechanics					
Moment of inertia, 2nd Moment of mass	J	kilogram x square meter	$kg \cdot m^2$	The following applies for a homogenous body: $J = \varrho \cdot r^2 \cdot V$	The moment of inertia (2nd moment of mass) is dependent upon the total mass of the body as well as its form and the position of the axis of rotation.
Force Weight	F F_G, G	newton	N	$1\ N = 1 \frac{kg \cdot m}{s^2} = 1 \frac{J}{m}$ $1\ MN = 10^3\ kN = 1\,000\,000\ N$	The force 1 N effects a change in velocity of 1 m/s in 1 s in a 1 kg mass.
Torque Bending mom. Torsional mom.	M M_b T	newton x meter	$N \cdot m$	$1\ N \cdot m = 1 \frac{kg \cdot m^2}{s^2}$	1 N · m is the moment that a force of 1 N effects with a lever arm of 1 m.
Momentum	p	kilogram x meter per second	$kg \cdot m/s$	$1\ kg \cdot m/s = 1\ N \cdot s$	The momentum is the product of the mass times velocity. It has the direction of the velocity.
Pressure Mechanical stress	p σ, τ	pascal newton per square millimeter	Pa N/mm^2	$1\ Pa = 1\ N/m^2 = 0.01\ mbar$ $1\ bar = 100\,000\ N/m^2 = 10\ N/cm^2 = 10^5\ Pa$ $1\ mbar = 1\ hPa$ $1\ N/mm^2 = 10\ bar = 1\ MN/m^2 = 1\ MPa$ $1\ daN/cm^2 = 0.1\ N/mm^2$	Pressure refers to the force per unit area. For gage pressure the symbol p_g is used (DIN 1314). 1 bar = 14.5 psi (pounds per square inch)
Second moment of area	I	meter to the fourth power centimeter to the fourth power	m^4 cm^4	$1\ m^4 = 100\,000\,000\ cm^4$	Previously: Geometrical moment of inertia
Energy, Work, Quantity of heat	E, W	joule	J	$1\ J = 1\ N \cdot m = 1\ W \cdot s = 1\ kg \cdot m^2/s^2$	Joule for all forms of energy, kW · h preferred for electrical energy.
Power Heat flux	P Φ	watt	W	$1\ W = 1\ J/s = 1\ N \cdot m/s = 1\ V \cdot A = 1\ m^2 \cdot kg/s^3$	Power describes the work which is achieved within a specific time.
Time					
Time, Time span, Duration	t	**seconds** minutes hours day year	s min h d a	$1\ min = 60\ s$ $1\ h = 60\ min = 3600\ s$ $1\ d = 24\ h = 86\,400\ s$	3 h means a time span (3 hrs.), 3^h means a point in time (3 o'clock). If points in time are written in mixed form, e.g. $3^h24^m10^s$, the symbol min can be shortened to m.
Frequency	f, ν	hertz	Hz	$1\ Hz = 1/s$	1 Hz ≙ 1 cycle in 1 second.
Rotational speed, Rotational frequency	n	1 per second 1 per minute	1/s 1/min	$1/s = 60/min = 60\ min^{-1}$ $1/min = 1\ min^{-1} = \frac{1}{60\ s}$	The number of revolutions per unit of time gives the revolution frequency, also called rpm.
Velocity	v	meters per second meters per minute kilometers per hour	m/s m/min km/h	$1\ m/s = 60\ m/min = 3.6\ km/h$ $1\ m/min = \frac{1\ m}{60\ s}$ $1\ km/h = \frac{1\ m}{3.6\ s}$	Nautical velocity in knots (kn): 1 kn = 1.852 km/h miles per hour = 1 mile/h = 1 mph 1 mph = 1.60934 km/h
Angular-velocity	ω	1 per second radians per second	1/s rad/s	$\omega = 2\,\pi \cdot n$	For a rpm of $n = 2/s$ the angular velocity $\omega = 4\,\pi/s$.
Acceleration	a, g	meters per second squared	m/s^2	$1\ m/s^2 = \frac{1\ m/s}{1\ s}$	Symbol g only for acceleration due to gravity. $g = 9.81\ m/s^2 \approx 10\ m/s^2$

Units of measurement

Quantities and units (continued)

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Electricity and Magnetism					
Electric current Electromotive force Electrical resistance Electrical conductance	I E R G	**ampere** volt ohm siemens	A V Ω S	1 V = 1 W/1 A = 1 J/C 1 Ω = 1 V/1 A 1 S = 1 A/1 V = 1/Ω	The motion of an electrical charge is called current. The electromotive force is equal to the potential difference between two points in an electric field. The reciprocal of the electrical resistance is called the electrical conductivity.
Specific resistance Conductivity	ϱ $\gamma, \varkappa$	ohm x meter siemens per meter	Ω · m S/m	$10^{-6}\ \Omega \cdot m = 1\ \Omega \cdot mm^2/m$	$\varrho = \frac{1}{\varkappa}$ in $\frac{\Omega \cdot mm^2}{m}$ $\varkappa = \frac{1}{\varrho}$ in $\frac{m}{\Omega \cdot mm^2}$
Frequency	f	hertz	Hz	1 Hz = 1/s 1000 Hz = 1 kHz	Frequency of public electric utility: EU 50 Hz, USA/Canada 60 Hz
Electrical energy	W	joule	J	1 J = 1 W · s = 1 N · m 1 kW · h = 3.6 MJ 1 W · h = 3.6 kJ	In atomic and nuclear physics the unit eV (electron volt) is used.
Phase difference	φ	–	–	for alternating current: $\cos \varphi = \frac{P}{U \cdot I}$	The angle between current and voltage in inductive or capacitive load.
Elect. field strength Elect. charge Elect. capacitance inductance	E Q C L	volts per meter coulomb farad henry	V/m C F H	1 C = 1 A · 1 s; 1 A · h = 3.6 kC 1 F = 1 C/V 1 H = 1 V · s/A	$E = \frac{F}{Q}$, $C = \frac{Q}{U}$, $Q = I \cdot t$
Power Effective power	P	watt	W	1 W = 1 J/s = 1 N · m/s = 1 V · A	In electrical power engineering: Apparent power S in V · A

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Electricity and Magnetism					
Thermodynamic temperature Celsius temperature	T, Θ t, ϑ	**kelvin** degrees Celsius	K °C	0 K = – 273.15 °C 0 °C = 273.15 K 0 °C = 32 °F 0 °F = – 17.77 °C	Kelvin (K) and degrees Celsius (°C) are used for temperatures and temperature differences. $t = T - T_0$; $T_0 = 273.15$ K degrees Fahrenheit (°F): 1.8 °F = 1 °C
Quantity of heat	Q	joule	J	1 J = 1 W · s = 1 N · m 1 kW · h = 3 600 000 J = 3,6 MJ	1 kcal ≙ 4.1868 kJ
Net calorific value	H_u	joule per kilogram Joule per cubic meter	J/kg J/m³	1 MJ/kg = 1 000 000 J/kg $1\ MJ/m^3 = 1\,000\,000\ J/m^3$	Thermal energy released per kg fuel minus the heat of vaporization of the water vapor contained in the exhaust gases.

Non-SI units

Length	Area	Volume	Mass	Energy, Power
1 inch (in) = 25.4 mm	1 sq.in = 6.452 cm²	1 cu.in = 16.39 cm³	1 oz = 28.35 g	1 PSh = 0.735 kWh
1 foot (ft) = 0.3048 m	1 sq.ft = 9.29 dm²	1 cu.ft = 28.32 dm³	1 lb = 453.6 g	1 PS = 735 W
1 yard (yd) = 0.9144 m	1 sq.yd = 0.8361 m²	1 cu.yd = 764.6 dm³	1 t = 1000 kg	1 kcal = 4186.8 Ws
1 nautical mile = 1.852 km	1 acre = 4046.856 m²	1 gallon (US) = 3.785 l	1 short ton = 907.2 kg	1 kcal = 1.166 Wh
1 mile = 1.6093 km	**Pressure**	1 gallon (UK) = 4.546 l	1 Karat = 0.2 g	1 kpm/s = 9.807 W
	1 bar = 14.5 pound/in²	1 barrel = 158.8 l	1 pound/in³ = 27.68 g/cm³	1 Btu = 1055 Ws
	1 N/mm² = 145.038 pound/in²			1 hp = 745.7 W

Formula symbols, Mathematical symbols

Formula symbols

cf. DIN 1304-1 (1994-03)

Formula symbol	Meaning	Formula symbol	Meaning	Formula symbol	Meaning
Length, Area, Volume, Angle					
l	Length	r, R	Radius	α, β, γ	Planar angle
w	Width	d, D	Diameter	Ω	Solid angle
h	Height	A, S	Area, Cross-sectional area	λ	Wave length
s	Linear distance	V	Volume		
Mechanics					
m	Mass	F	Force	G	Shear modulus
m'	Linear mass density	F_W, W	Gravitational force, Weight	μ, f	Coefficient of friction
m''	Area mass density	M	Torque	W	Section modulus
ϱ	Density	T	Torsional moment	I	Second moment of an area
J	Moment of inertia	M_b	Bending moment	W, E	Work, Energy
p	Pressure	σ	Normal stress	W_p, E_p	Potential energy
p_{abs}	Absolute pressure	τ	Shear stress	W_k, E_k	Kinetic energy
p_{amb}	Ambient pressure	ε	Normal strain	P	Power
p_g	Gage pressure	E	Modulus of elasticity	η	Efficiency
Time					
t	Time, Duration	f, ν	Frequency	a	Acceleration
T	Cycle duration	v, u	Velocity	g	Gravitational acceleration
n	Revolution frequency, Speed	ω	Angular velocity	α	Angular acceleration
				$Q, \dot{V}, q_v$	Volumetric flow rate
Electricity					
Q	Electric charge, Quantity of electricity	L	Inductance	X	Reactance
E	Electromotive force	R	Resistance	Z	Impedance
C	Capacitance	ϱ	Specific resistance	φ	Phase difference
I	Electric current	$\gamma, \varkappa$	Electrical conductivity	N	Number of turns
Heat					
T, Θ	Thermodynamic temperature	Q	Heat, Quantity of heat	$\Phi, \dot{Q}$	Heat flow
$\Delta T, \Delta t, \Delta\vartheta$	Temperature difference	λ	Thermal conductivity	a	Thermal diffusivity
t, ϑ	Celsius temperature	α	Heat transition coefficient	c	Specific heat
α_l, α	Coefficient of linear expansion	k	Heat transmission coefficient	H_{net}	Net calorific value
Light, Electromagnetic radiation					
E	Illuminance	f	Focal length	I	Luminous intensity
		n	Refractive index	Q, W	Radiant energy
Acoustics					
p	Acoustic pressure	L_p	Acoustic pressure level	N	Loudness
c	Acoustic velocity	I	Sound intensity	L_N	Loudness level

Mathematical symbols

cf. DIN 1302 (1999-12)

Math. symbol	Spoken	Math. symbol	Spoken	Math. symbol	Spoken
$\approx$	approx. equals, around, about	$\sim$	proportional	log	logarithm (general)
$\hat{=}$	equivalent to	a^n	a to the n-th power, the n-th power of a	lg	common logarithm
$\ldots$	and so on, etc.	$\sqrt{\ }$	square root of	ln	natural logarithm
∞	infinity	$\sqrt[n]{\ }$	n-th root of	e	Euler number (e = 2.718281…)
$=$	equal to	$\lvert x \rvert$	absolute value of x	sin	sine
$\neq$	not equal to	$\perp$	perpendicular to	cos	cosine
$\overset{def}{=}$	is equal to by definition	$\parallel$	is parallel to	tan	tangent
$<$	less than	$\uparrow\uparrow$	parallel in the same direction	cot	cotangent
$\leq$	less than or equal to	$\uparrow\downarrow$	parallel in the opposite direction	(), [], { }	parentheses, brackets open and closed
$>$	greater than	$\sphericalangle$	angle	π	pi (circle constant = 3.14159 …)
$\geq$	greater than or equal to	$\triangle$	triangle		
$+$	plus	$\cong$	congruent to		
$-$	minus	Δx	delta x (difference between two values)	$\overline{AB}$	line segment AB
$\cdot$	times, multiplied by	%	percent, of a hundred	$\overset{\frown}{AB}$	arc AB
$-, /, :, \div$	over, divided by, per, to	‰	per mil, of a thousand	a', a''	a prime, a double prime
Σ	sigma (summation)			a_1, a_2	a sub 1, a sub 2

M

Formulas, Equations, Graphs

Formulas

In most cases, the calculation of physical quantities is done with the help of formulas. They consist of:

- Formula symbols, e.g. v_c for cutting velocity, d for diameter, n for speed
- Operators (calculation rules), e.g. · for multiplication, + for addition, – for subtraction and — (fraction line) for division
- Constants, e.g. π (pi) = 3.14159 …
- Numbers, e.g. 10, 15 …

The formula symbols (page 13) are wildcards for quantities. When solving mathematical problems, the known quantities with their units are filled in the formulas. Before or during the calculation process, the units are converted in a way that

- the calculation becomes feasible or
- the result comprises the required unit.

Most quantities and units are standardized (page 10).

The **result** is always a **numerical value** accompanied by a **unit**, e.g. 4.5 m, 15 s

Example:

What is the cutting velocity v_c in m/min for d = 200 mm and n = 630/min?

$$v_c = \pi \cdot d \cdot n = \pi \cdot 200\ \text{mm} \cdot 630\ \frac{1}{\text{min}} = \pi \cdot 200\ \text{mm} \cdot \frac{1\ \text{m}}{1000\ \text{mm}} \cdot 630\ \frac{1}{\text{min}} = \mathbf{395.84}\ \frac{\mathbf{m}}{\mathbf{min}}$$

Formula for cutting velocity

$$v_c = \pi \cdot d \cdot n$$

Numerical value equations

Numerical value equations or numerical equations are formulas in which the typical conversions of units have already been integrated. The following should be noted when using equations:

The numerical values of the individual quantities may only be used in combination with the designated unit.

- The units are not carried along in the calculation.
- The unit of the quantity to be obtained is predetermined.

Example:

What is the torque M of an electrical motor with a driving power of P = 15 kW and a speed of n = 750/min?

$$M = \frac{9550 \cdot P}{n} = \frac{9550 \cdot 15}{750}\ \text{N} \cdot \text{m} = \mathbf{191\ N \cdot m}$$

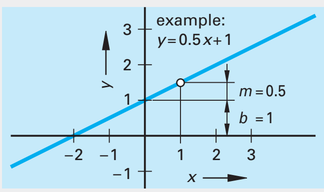
Numerical value equation for torque

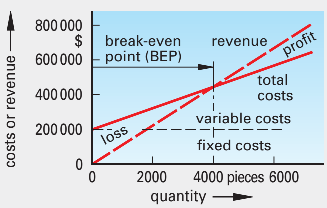
$$M = \frac{9550 \cdot P}{n}$$

Designated unit		
Designation		Unit
M	Torque	N · m
P	Power	kW
n	Speed	1/min

Equations and graphs

In functional equations, y is the function of x, with x as an independent and y as a dependent variable. The number pairs (x, y) of a value table form a graph in the x-y system of coordinates.





1st example:

$y = 0.5\ x + 1$

x	–2	0	2	3
y	0	1	2	2.5

2nd example:

Cost function and revenue function

C_t = 60 \$/piece · Q + 200 000 \$

R = 110 \$/piece · Q

Q	0	4 000	6 000
C_t	200 000	440 000	560 000
R	0	440 000	660 000

C_t total costs → dependent variable
Q quantity → independent variable
C_f fixed costs → y coordinate section
C_v variable costs → gradient of the function
R revenue → dependent variable

Assigned function

$$y = f(x)$$

Linear function

$$y = m \cdot x + b$$

Examples:

Cost function

$$C_t = C_V \cdot Q + C_f$$

Revenue function

$$R = R/\text{piece} \cdot Q$$

M

Transformation of formulas

Transformation of formulas

Formulas and numerical equations are transformed so that the quantity to be obtained stands alone on the left side of the equation. The value of the left side and right side of the formula must not change during the transformation. The following rule applies to all steps of the formula transformation.

Changes applied to the left formula side	=	Changes applied to the right formula side

To be able to trace each step of the transformation, it is useful to mark it to the right next to the formula:

$| \cdot t$ → both sides of the formula are multiplied by t.

$| : F$ → both sides of the formula are divided by F.

Formula

$$P = \frac{F \cdot s}{t}$$

left side of the formula = right side of the formula

Transformations of sums

Example: formula $L = l_1 + l_2$, transformation to find l_2

1 $L = l_1 + l_2$	$\| - l_1$	subtract l_1	3 $L - l_1 = l_2$	invert both sides
2 $L - l_1 = l_1 + l_2 - l_1$		perform subtraction	4 $l_2 = L - l_1$	transformed formula

Transformations of products

Example: formula $A = l \cdot b$, transformation to find l

1 $A = l \cdot b$	$\| : b$	divide by b	3 $\frac{A}{b} = l$	invert both sides
2 $\frac{A}{b} = \frac{l \cdot b}{b}$		cancel b	4 $l = \frac{A}{b}$	transformed formula

Transformations of fractions

Example: formula $n = \frac{l}{l_1 + s}$, transformation to find s

1 $n = \frac{l}{l_1 + s}$	$\| \cdot (l_1 + s)$	multiply by $(l_1 + s)$	4 $n \cdot l_1 - n \cdot l_1 + n \cdot s = l - n \cdot l_1$	$\| : n$	subtract divide by n
2 $n \cdot (l_1 + s) = \frac{l \cdot (l_1 + s)}{(l_1 + s)}$		cancel $(l_1 + s)$ on the right side solve the term in brackets	5 $\frac{s \cdot n}{n} = \frac{l - n \cdot l_1}{n}$		cancel n
3 $n \cdot l_1 + n \cdot s = l$	$\| - n \cdot l_1$	subtract $- n \cdot l_1$	6 $s = \frac{l - n \cdot l_1}{n}$		transformed formula

Transformations of roots

Example: formula $c = \sqrt{a^2 + b^2}$, transformation to find a

1 $c = \sqrt{a^2 + b^2}$	$\| ()^2$	square equation	4 $a^2 = c^2 - b^2$	$\| \sqrt{\ }$	extract the root
2 $c^2 = a^2 + b^2$	$\| - b^2$	subtract b^2	5 $\sqrt{a^2} = \sqrt{c^2 - b^2}$		simplify the expression
3 $c^2 - b^2 = a^2 + b^2 - b^2$		subtract, invert both sides	6 $a = \sqrt{c^2 - b^2}$		transformed formula

Quantities and units

Numerical values and units

Physical quantity

10 mm

Numerical value — Unit

Physical quantities, e.g. 125 mm, consist of a

- **numerical value,** which is determined by measurement or calculation, and a
- **unit,** e.g. m, kg

Units are standardized in accordance with DIN 1301-1 (page 10).

Very large or very small numerical values may be represented in a simplified way as decimal multiples or factors with the help of prefixes, e.g. 0.004 mm = 4 µm.

Decimal multiples or factors of units

cf. DIN 1301-1 (2004-10)

Prefix		Power of ten	Mathematical designation	Examples
Symbol	Name			
T	tera	10^{12}	trillion	12 000 000 000 000 N = $12 \cdot 10^{12}$ N = 12 TN (teranewtons)
G	giga	10^{9}	billion	45 000 000 000 W = $45 \cdot 10^{9}$ W = 45 GW (gigawatts)
M	mega	10^{6}	million	8 500 000 V = $8.5 \cdot 10^{6}$ V = 8.5 MV (megavolts)
k	kilo	10^{3}	thousand	12 600 W = $12.6 \cdot 10^{3}$ W = 12.6 kW (kilowatts)
h	hecto	10^{2}	hundred	500 l = $5 \cdot 10^{2}$ l = 5 hl (hectoliters)
da	deca	10^{1}	ten	32 m = $3.2 \cdot 10^{1}$ m = 3.2 dam (decameters)
–	–	10^{0}	one	1.5 m = $1.5 \cdot 10^{0}$ m
d	deci	10^{-1}	tenth	0.5 l = $5 \cdot 10^{-1}$ l = 5 dl (deciliters)
c	centi	10^{-2}	hundredth	0.25 m = $25 \cdot 10^{-2}$ m = 25 cm (centimeters)
m	milli	10^{-3}	thousandth	0.375 A = $375 \cdot 10^{-3}$ A = 375 mA (milliamperes)
µ	micro	10^{-6}	millionth	0.000 052 m = $52 \cdot 10^{-6}$ m = 52 µm (micrometers)
n	nano	10^{-9}	billionth	0.000 000 075 m = $75 \cdot 10^{-9}$ m = 75 nm (nanometers)
p	pico	10^{-12}	trillionth	0.000 000 000 006 F = $6 \cdot 10^{-12}$ F = 6 pF (picofarads)

Conversion of units

Calculations with physical units are only possible if these units refer to the same base in this calculation. When solving mathematical problems, units often must be converted to basic units, e. g. mm to m, s to h, mm^2 to m^2. This is done with the help of conversion factors that represent the value 1 (coherent units).

Conversion factors for units (excerpt)

Quantity	Conversion factors, e. g.	Quantity	Conversion factors, e. g.
Length	$1 = \frac{10\ \text{mm}}{1\ \text{cm}} = \frac{1000\ \text{mm}}{1\ \text{m}} = \frac{1\ \text{m}}{1000\ \text{mm}} = \frac{1\ \text{km}}{1000\ \text{m}}$	Time	$1 = \frac{60\ \text{min}}{1\ \text{h}} = \frac{3600\ \text{s}}{1\ \text{h}} = \frac{60\ \text{s}}{1\ \text{min}} = \frac{1\ \text{min}}{60\ \text{s}}$
Area	$1 = \frac{100\ \text{mm}^2}{1\ \text{cm}^2} = \frac{100\ \text{cm}^2}{1\ \text{dm}^2} =$	Angle	$1 = \frac{60'}{1°} = \frac{60''}{1'} = \frac{3600''}{1°} = \frac{1°}{60\ \text{s}}$
Volume	$1 = \frac{1000\ \text{mm}^3}{1\ \text{cm}^3} = \frac{1000\ \text{cm}^3}{1\ \text{dm}^3} =$	Inch	1 inch = 25.4 mm; 1 mm = $\frac{1}{25.4}$ inches

1st example:

Convert volume $V = 3416\ \text{mm}^3$ to cm^3.

Volume V is multiplied by a conversion factor. Its numerator has the unit cm^3 and its denominator the unit mm^3.

$$\boldsymbol{V} = 3416\ mm^3 = \frac{1\ cm^3 \cdot 3416\ mm^3}{1000\ mm^3} = \frac{3416\ cm^3}{1000} = \mathbf{3.416\ cm^3}$$

2nd example:

The angle size specification $\alpha = 42°\,16'$ is to be expressed in degrees (°).

The partial angle 16′ must be converted to degrees (°). The value is multiplied by a conversion factor, the numerator of which has the unit degree (°) and the denominator the unit minute (′).

$$\boldsymbol{\alpha} = 42° + 16' \cdot \frac{1°}{60'} = 42° + \frac{16 \cdot 1°}{60} = 42° + 0.267° = \mathbf{42.267°}$$

Calculation with quantities, Percentage and interest calculation

M

Calculation with quantities

Physical quantities are mathematically treated as products.

- **Adding and subtracting**

  Numerical values that have the same unit are added or subtracted and the unit is carried over to the result.

Example:

$L = l_1 + l_2 - l_3$ mit $l_1 = 124$ mm, $l_2 = 18$ mm, $l_3 = 44$ mm; L = ?

$\boldsymbol{L} = 124\text{ mm} + 18\text{ mm} - 44\text{ mm} = (124 + 18 - 44)\text{ mm} = \mathbf{98\ mm}$

- **Multiplying and dividing**

  The numerical values and the units correspond to the factors of products.

Example:

$F_1 \cdot l_1 = F_2 \cdot l_2$ mit $F_1 = 180$ N, $l_1 = 75$ mm, $l_2 = 105$ mm; F_2 = ?

$$\boldsymbol{F_2} = \frac{F_1 \cdot l_1}{l_2} = \frac{180\text{ N} \cdot 75\text{ mm}}{105\text{ mm}} = 128.57\ \frac{\text{N} \cdot \text{mm}}{\text{mm}} = \mathbf{128.57\ N}$$

- **Multiplying and dividing powers**

  Powers that have the same base are multiplied or divided by adding or subtracting their exponents.

Example:

$W = \frac{A \cdot a^2}{e}$ with $A = 15$ cm², $a = 7.5$ cm, $e = 2.4$ cm; W = ?

$$\boldsymbol{W} = \frac{15\text{ cm}^2 \cdot (7.5\text{ cm})^2}{2.4\text{ cm}} = \frac{15 \cdot 56.25\text{ cm}^{2+2}}{2.4\text{ cm}^1} = 351.56\text{ cm}^{4-1} = \mathbf{351.56\ cm^3}$$

Rules for raising to powers

a base

m, n … exponents

Multiplying powers

$$a^2 \cdot a^3 = a^{2+3}$$

Dividing powers

$$\frac{a^2}{a^3} = a^{2-3}$$

Special cases

$$a^{-2} = \frac{1}{a^2}$$

$$a^1 = a \qquad a^0 = 1$$

Percentage calculation

The **percentage rate** indicates the part of the base value in hundredths.
The **base value** is the value from which the percentage is to be calculated.
The **percent value** is the amount representing the percentage of the base value.

P_r percentage rate, in percent $\quad$ P_v percent value $\quad$ B_v base value

Example:

Weight of raw part: 250 kg (base value); material loss of 2 % (percentage rate); material loss in kg = ? (percent value)

$$\boldsymbol{P_v} = \frac{B_v \cdot P_r}{100\,\%} = \frac{250\text{ kg} \cdot 2\,\%}{100\,\%} = \mathbf{5\ kg}$$

Percent value

$$P_v = \frac{B_v \cdot P_r}{100\,\%}$$

Interest calculation

P principle
A amount accumulated
I interest
r interest rate per year
t period in days, interest period

1st Example:

$P = \$\,2800.00;\ r = 6\,\frac{\%}{a};\ t = \frac{1}{2}\text{ a};\ I = ?$

$$I = \frac{\$\,2800.00 \cdot 6\,\frac{\%}{a} \cdot 0.5\text{ a}}{100\,\%} = \$\,84.00$$

2nd Example:

$P = \$\,4800.00;\ r = 5.1\,\frac{\%}{a};\ t = 50\text{ d};\ I = ?$

$$I = \frac{\$\,4800.00 \cdot 5.1\,\frac{\%}{a} \cdot 50\text{ d}}{100\,\% \cdot 360\,\frac{d}{a}} = \$\,34.00$$

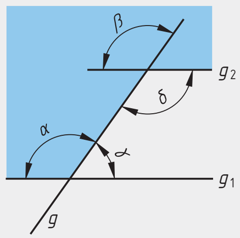
Interest

$$I = \frac{P \cdot r \cdot t}{100\,\% \cdot 360}$$

1 interest year (1 a) = 360 days (360 d)
360 d = 12 months
1 interest month = 30 days

Types of angles, Theorem of intersecting lines, Angles in a triangle, Pythagorean theorem

Types of angles



g	straight line
g_1, g_2	parallel straight lines
α, β	corresponding angles
β, δ	opposite angles
α, δ	alternate angles
α, γ	adjacent angles

If two parallels are intersected by a straight line, there are geometrical interrelationships between the resulting angles.

Corresponding angles

$$\alpha = \beta$$

Opposite angles

$$\beta = \delta$$

Alternate angles

$$\alpha = \delta$$

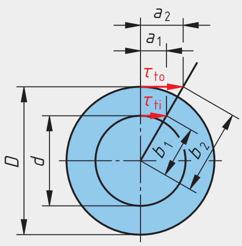
Adjacent angles

$$\alpha + \gamma = 180°$$

Theorem of intersecting lines

τ_{to} outer torsional stress

τ_{ti} inner torsional stress



If two intersecting lines are intercepted by a pair of parallels, the resulting segments form equal ratios.

Example:

D = 40 mm, d = 30 mm,
τ_{ta} = 135 N/mm²; τ_{ti} = ?

$$\frac{\tau_{ti}}{\tau_{to}} = \frac{d}{D} \Rightarrow \tau_{ti} = \frac{\tau_{to} \cdot d}{D}$$

$$= \frac{135 \text{ N/mm}^2 \cdot 30 \text{ mm}}{40 \text{ mm}} = \mathbf{101.25 \text{ N/mm}^2}$$

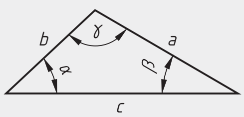
Theorem of intersecting lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{\frac{d}{2}}{\frac{D}{2}}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\frac{b_1}{d} = \frac{b_2}{D}$$

Sum of angles in a triangle



a, b, c	sides of the triangle
α, β, γ	angles in the triangle

Example:

α = 21°, β = 95°, γ = ?

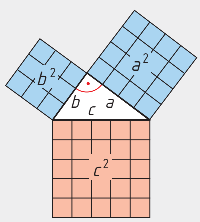
$\boldsymbol{\gamma}$ = 180° – α – β = 180° – 21° – 95° = **64°**

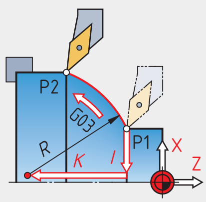
Sum of angles in a triangle

$$\alpha + \beta + \gamma = 180°$$

In every triangle, the sum of the interior angles equals 180°.

Pythagorean theorem





In a **right triangle** the square on the hypotenuse is equal to the sum of the squares on the sides meeting the right angle.

a	side
b	side
c	hypotenuse

1st example:

c = 35 mm; a = 21 mm; b = ?

$\mathbf{b} = \sqrt{c^2 - a^2} = \sqrt{(35 \text{ mm})^2 - (21 \text{ mm})^2} = \mathbf{28 \text{ mm}}$

2nd example:

CNC programm with R = 50 mm and I = 25 mm.
K = ?

$c^2 = a^2 + b^2$

$R^2 = I^2 + K^2$

$K = \sqrt{R^2 - I^2} = \sqrt{50^2 \text{ mm}^2 - 25^2 \text{ mm}^2}$

$\mathbf{K}$ = **43.3 mm**

Length of the hypotenuse

$$c^2 = a^2 + b^2$$

Square on the hypotenuse

$$c = \sqrt{a^2 + b^2}$$

Length of the sides meeting the right angle

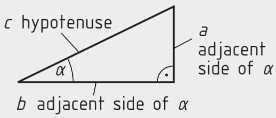
$$a = \sqrt{c^2 - b^2}$$

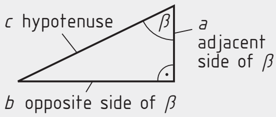
$$b = \sqrt{c^2 - a^2}$$

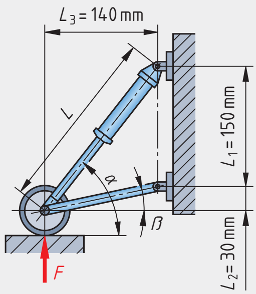
M

Functions of triangles

Functions of right triangles (trigonometric functions)







c	hypotenuse (longest side)
a, b	sides, – b is the adjacent side of α – a is the opposite side of α
α, β, γ	angles in the triangle, $\gamma = 90°$
sin	notation of sine
cos	notation of cosine
tan	notation of tangent
$\sin\alpha$	sine of angle α

1st example

L_1 = 150 mm, L_2 = 30 mm, L_3 = 140 mm; angle α = ?

$$\tan\alpha = \frac{L_1 + L_2}{L_3} = \frac{180\text{ mm}}{140\text{ mm}} = 1.286$$

Angle α = 52°

2nd example

L_1 = 150 mm, L_2 = 30 mm, α = 52°; Length of the shock absorber L = ?

$$L = \frac{L_1 + L_2}{\sin\alpha} = \frac{180\text{ mm}}{\sin 52°} = \mathbf{228.42\text{ mm}}$$

Trigonometric functions

sine	=	$\frac{\text{opposite side}}{\text{hypotenuse}}$
cosine	=	$\frac{\text{adjacent side}}{\text{hypotenuse}}$
tangent	=	$\frac{\text{opposite side}}{\text{adjacent side}}$

Relations applying to angle α:

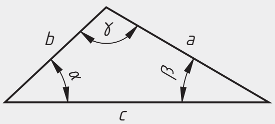
$\sin\alpha = \frac{a}{c}$	$\cos\alpha = \frac{b}{c}$	$\tan\alpha = \frac{a}{b}$

Relations applying to angle β:

$\sin\beta = \frac{b}{c}$	$\cos\beta = \frac{a}{c}$	$\tan\beta = \frac{b}{a}$

The calculation of an angle in degrees (°) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arcsine.

Functions of oblique triangles (law of sines, law of cosines)



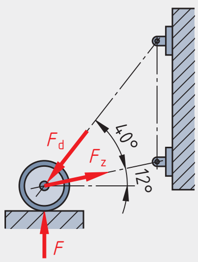
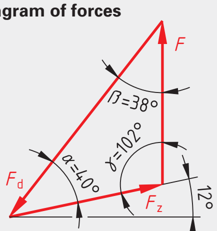


Diagram of forces



According to the law of sines, the ratios of the sides correspond to the sine of their opposite angles in the triangle. If one side and two angles are known, the other values can be calculated with the help of this function.

Side a → opposite angle $\sin\alpha$
Side b → opposite angle $\sin\beta$
Hypothenuse c → opposite angle $\sin\gamma$

Example:

F = 800 N, α = 40°, β = 38°; F_z = ?, F_d = ?

The forces are calculated with the help of the forces diagram.

$$\frac{F}{\sin\alpha} = \frac{F_z}{\sin\beta} \Rightarrow F_z = \frac{F \cdot \sin\beta}{\sin\alpha}$$

$$F_z = \frac{800\text{ N} \cdot \sin 38°}{\sin 40°} = \mathbf{766.24\text{ N}}$$

$$\frac{F}{\sin\alpha} = \frac{F_d}{\sin\gamma} \Rightarrow F_d = \frac{F \cdot \sin\gamma}{\sin\alpha}$$

$$F_d = \frac{800\text{ N} \cdot \sin 102°}{\sin 40°} = \mathbf{1217.38\text{ N}}$$

The calculation of an angle in degrees (°) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arcsine.

Law of sines

$$a : b : c = \sin\alpha : \sin\beta : \sin\gamma$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

There are many transformation options:

$$a = \frac{b \cdot \sin\alpha}{\sin\beta} = \frac{c \cdot \sin\alpha}{\sin\gamma}$$

$$b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{c \cdot \sin\beta}{\sin\gamma}$$

$$c = \frac{a \cdot \sin\gamma}{\sin\alpha} = \frac{b \cdot \sin\gamma}{\sin\beta}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$$
$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$$
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$$

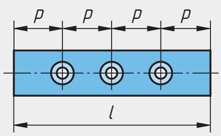
Transformation, e. g.

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$$

Division of lengths, Arc length, Composite length

Sub-dividing lengths

Edge distance = spacing



l total length
p spacing
n number of holes

Example:

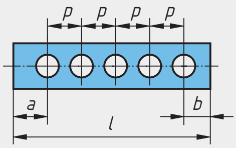
$l = 2$ m; $n = 24$ holes; $p = ?$

$p = \frac{l}{n+1} = \frac{2000\text{ mm}}{24+1} = \mathbf{80\,mm}$

Spacing

$$p = \frac{l}{n+1}$$

Edge distance ≠ spacing



l total length
p spacing
n number of holes
a, b edge distances

Example:

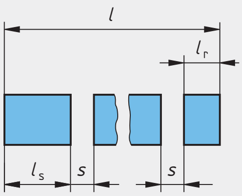
$l = 1950$ mm; $a = 100$ mm; $b = 50$ mm;
$n = 25$ holes; $p = ?$

$p = \frac{l-(a+b)}{n-1} = \frac{1950\text{ mm} - 150\text{ mm}}{25-1} = \mathbf{75\,mm}$

Spacing

$$p = \frac{l-(a+b)}{n-1}$$

Subdividing into pieces



l bar length
z number of pieces
l_s piece length
s saw cutting width
l_r remaining length

Example:

$l = 6000$ mm; $l_s = 230$ mm; $s = 1.2$ mm; $z = ?$; $l_r = ?$

$z = \frac{l}{l_s + s} = \frac{6000\text{ mm}}{230\text{ mm} + 1.2\text{ mm}} = \mathbf{25.95 = 25\ pieces}$

$l_r = l - z \cdot (l_s + s) = 6000\text{ mm} - 25 \cdot (230\text{ mm} + 1.2\text{ mm})$
$= \mathbf{220\,mm}$

Number of pieces

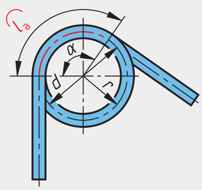
$$z = \frac{l}{l_s + s}$$

Remaining length

$$l_r = l - z \cdot (l_s + s)$$

Arc length

Example: Torsion spring



l_a arc length
r radius
α angle at center
d diameter

Example:

$r = 36$ mm; $\alpha = 120°$; $l_a = ?$

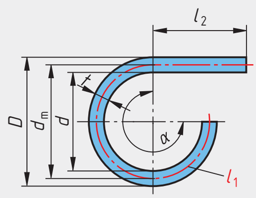
$l_a = \frac{\pi \cdot r \cdot \alpha}{180°} = \frac{\pi \cdot 36\text{ mm} \cdot 120°}{180°} = \mathbf{75.36\,mm}$

Arc length

$$l_a = \frac{\pi \cdot r \cdot \alpha}{180°}$$

$$l_a = \frac{\pi \cdot d \cdot \alpha}{360°}$$

Composite length



D outside diameter
d_m mean diameter
l_1, l_2 section lengths
α angle at center
d inside diameter
t thickness
L composite length

Example (composite length, picture left):

$D = 360$ mm; $t = 5$ mm; $\alpha = 270°$; $l_2 = 70$ mm;
$d_m = ?$; $L = ?$

$\mathbf{d_m} = D - t = 360\text{ mm} - 5\text{ mm} = \mathbf{355\,mm}$

$\mathbf{L} = l_1 + l_2 = \frac{\pi \cdot d_m \cdot \alpha}{360} + l_2$

$= \frac{\pi \cdot 355\text{ mm} \cdot 270°}{360°} + 70\text{ mm} = \mathbf{906.45\,mm}$

Composite length

$$L = l_1 + l_2 + \ldots$$